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ANALYSIS OF THE GENERAL ELECTRIC MONOPULSE CLUTTER REJECTION TECHNIQUE

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ANALYSIS OF THE GENERAL ELECTRIC MONOPULSE CLUTTER REJECTION TECHNIQUE

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ABSTRACT

The ~~GE~~ "monopulse" technique attempts to obtain signal-to-clutter (S/C) improvement on point targets in distributed clutter by processing returns from two monopulse antenna beams (over and above the S/C improvement due to conventional MTI techniques). A theoretical upper bound on S/C improvement is derived here for the general class of systems which process returns from multiple beam positions. The maximum possible improvement is small, being only 1.25 db for a $\sin x/x$ antenna pattern and 2.85 db for a $\sin^2 x/x^2$ antenna pattern. GE's own computations, when correctly interpreted, indicate an improvement of from 0.3 db to 0.6 db (based on a $\sin x/x$ antenna pattern), in agreement with the theoretical upper bound obtained here.

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I. INTRODUCTION

General Electric has proposed the use of a discriminant for distinguishing targets from clutter, other than the conventional one of amplitude ratio (target-to-cancelled clutter).[1,2] This concept is based on the use of a monopulse antenna and will be referred to as "monopulse clutter rejection" (or simply "monopulse").[3]

The GE monopulse concept may be explained in terms of either amplitude-comparison monopulse, Fig. 1a, or phase-comparison monopulse, Fig. 1b. In the amplitude-comparison case, two separate, but somewhat overlapping, narrow beams are obtained from two slightly displaced feed horns of a parabolic reflector. A cross-over at the 3 db points may be assumed. A point target produces co-phasal returns A and B in the two beams. It follows that the sum, or Σ, output (A+B) will be co-phasal with the difference, or Δ, output (A-B), as shown in Fig. 1a. That is, Σ and Δ will be either exactly in phase, or exactly 180° out of phase.

The situation for clutter is considerably changed since each beam looks at a "different" clutter patch. While the two

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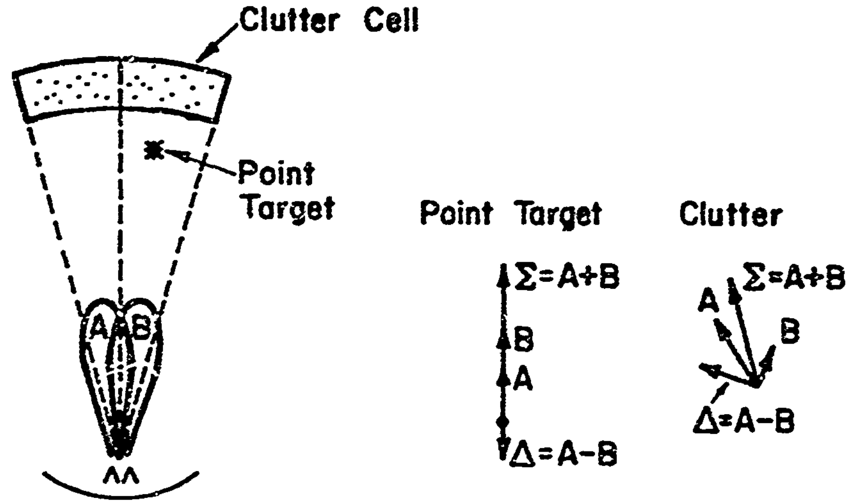
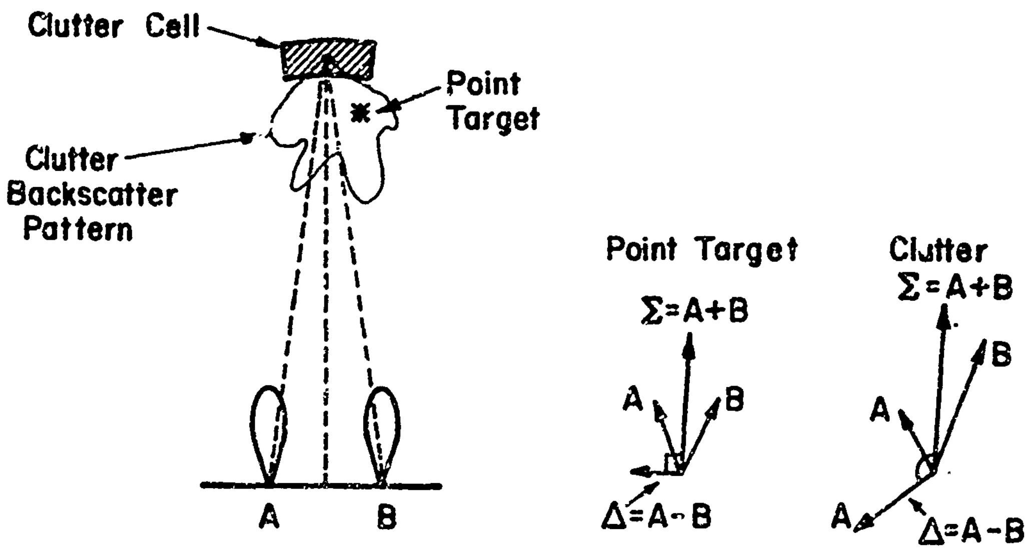
[1] W. Hausz, "Monopulse as an Aid to Clutter Rejection," 67TMP-56, TEMPO, General Electric Co., Santa Barbara, Calif., 1 June 1967.

[2] "Study for Application of Polydimensional Decision Techniques to AEW Radar," 11-006-7U, General Electric Co., Aerospace Electronics Dept., French Road, Utica, N. Y., October 1967.

[3] Only the azimuth monopulse concept is referred to here. The elevation monopulse concept, briefly mentioned by GE, is of an entirely different nature and has not been elaborated on by GE to where any comments could be made.



(a) Amplitude Comparison Monopulse



(b) Phase Comparison Monopulse

Fig. 1 GE MONOPULSE CLUTTER CANCELLATION CONCEPTS

beams overlap to some extent, it is a fair approximation to say that the two clutter returns will be almost statistically independent (i.e., their amplitudes and phases will be almost unrelated). This destroys the co-phasal relationship between Σ and Δ signals, which held for point targets. Only by mere chance would a co-phasal relationship result for clutter. Since a co-phasal condition is a low probability event for clutter but a high probability event for targets, GE proposes to measure the relative phase of Σ and Δ outputs to determine whether a target is present. Of course, a reasonably high S/C ratio would be necessary to accomplish this, but hopefully, there would be a net improvement over prior techniques in the ability to distinguish targets from clutter.

In the phase-comparison monopulse case, Fig. 1b, two arrays have identical beam patterns but widely separated phase centers. A point target produces essentially equal amplitudes in the two beams but a phase difference approximately proportional to angle off boresight. From Fig. 1b it is seen that Σ and Δ outputs are exactly in phase quadrature for a point target.

In the phase-comparison monopulse case both beams look at the same clutter cell. There is a difference in their received signals, however, based on the fact that the separation of their phase centers causes them to view the clutter cell from slightly different aspects. The clutter cell is a collection of randomly located scatterers and will have a random, finely lobed, back-scattering pattern, as indicated in Fig. 1b. The

difference in aspect for the two arrays causes them to be on significantly separated parts of the clutter backscatter pattern (on the order of one lobe separation), as shown in Appendix A, and therefore to receive signals of almost unrelated amplitude and phase. As indicated in Fig. 1b, this destroys the quadrature phase relationship between Σ and Δ outputs, which held for point targets. Again, since only by chance would a quadrature relationship result for clutter, this phase condition is a low probability event for clutter but a high probability event for point targets. In a method analogous to the amplitude comparison monopulse case, GE proposes to measure the relative phase of Σ and Δ outputs to establish whether or not a target is present.

While the GE monopulse technique can be used in a system which also employs MTI techniques, basically the GE technique has nothing to do with moving target discrimination. It is a method of discriminating point targets (not moving targets) from distributed clutter. Its potential applicability to the MTI problem lies in the fact that an aircraft target is a reasonable approximation to a point target, at least in comparison to the much larger clutter cell. Thus GE would obtain the clutter rejection capability of the monopulse technique in addition to the existing clutter rejection capabilities of conventional MTI techniques.

The problem of discriminating point targets from clutter has been investigated by Urkowitz[4] who determined the optimum

[4] H. Urkowitz, "Filters for the Detection of Small Radar Signals in Noise," Jour. Appl. Phys., Vol. 24, August 1953, pp. 1024-1031.

solution, the so-called "matched clutter filter." The analysis in the following section shows that conventional radar design, using amplitude thresholds only to discriminate point targets from clutter, is already very close to optimum in the Urkowitz sense. It follows that the GE monopulse clutter rejection technique can give little improvement over the conventional use of MTI technique followed by amplitude discrimination.

II. ANALYSIS

Since a direct analysis of the GE monopulse technique would require a large amount of computer time, we have taken the indirect approach of showing that the GE technique is inferior to the optimum processor, the "matched clutter filter" (MCF), and that the MCF itself is only 1 or 2 db better than no processor at all. Thus, the maximum possible improvement provided by the GE technique will be seen to be limited to 1 or 2 db.

The GE monopulse technique utilizes two displaced beams separated by an angle $\Delta\theta$. If one beam is at angle θ_1, the second beam is at angle $\theta_2 = \theta_1 + \Delta\theta$. Now consider a single beam rotating in angle. The single beam passes through the angles θ_1 and θ_2 in succession, as well as through all other angles. Comparing the data gathered by the two monopulse beams (assumed non-rotating) and the single, rotating beam, it is clear that the rotating beam output contains all the information provided by the two monopulse beams, and more. Thus, if the output of the single, rotating beam can be processed in an optimum manner, it will set an upper bound on performance which cannot be exceeded by the GE monopulse technique.

In Appendix B, the optimum filter for processing the output of the rotating antenna is shown to be a form of Urkowitz's "matched clutter filter." Now a matched filter is an absolutely optimum processor when the "noise" is gaussian. Random, distributed clutter, which is the "noise" source assumed in the present analysis, is gaussian. Thus the matched clutter filter

is not only the best filter, but the best of all possible processors. The performance of such a filter sets a true upper bound on system performance which cannot be exceeded by the GE monopulse technique.

As derived in Appendix B, the signal-to-clutter (power) ratio out of the MCF if

$$\frac{\left(P_s\right)_o}{\left(P_c\right)_o} = 2\,\frac{\sigma_t}{\sigma\rho}\,(L/\lambda),$$

where σ_t is target echo power, σ is the echo power of a single clutter element (both based on unity antenna gain), ρ is the number of clutter elements per radian, and L/λ is the ratio of antenna aperture length to RF wavelength. The above result is <u>independent of the aperture excitation</u> provided that the full aperture L is used.

With no filtering or processing, the signal-to-clutter ratio is shown in Appendix B to be

$$\frac{\left(P_s\right)_i}{\left(P_c\right)_i} = \frac{\sigma_t}{\sigma\rho}\;\frac{1}{\int_{-\infty}^{\infty} G_n^2(\phi)\,d\phi},$$

where $G_n(\phi)$ is the normalized one-way power pattern of the antenna and the target is assumed to be at the peak of the pattern.

The signal-to-clutter improvement factor $\left(I_{S/C}\right)$ due to matched clutter filtering is obtained as the ratio of the above equations:

$$I_{S/C} = \frac{\left(P_s\right)_o / \left(P_c\right)_o}{\left(P_s\right)_i / \left(P_c\right)_i} = 2(L/\lambda) \int_{-\infty}^{\infty} G_n^2(\psi)\, d\psi \; .$$

For a uniformly illuminated aperture,

$$G_n(\phi) = \frac{\sin^2(\pi \frac{L}{\lambda} \sin \phi)}{(\pi \frac{L}{\lambda} \sin \phi)^2} \approx \frac{\sin^2(\pi \frac{L}{\lambda} \phi)}{(\pi \frac{L}{\lambda} \phi)^2} \; , \; *$$

one obtains an improvement in signal-to-clutter of

$$I_{S/C} = \frac{4}{3} \text{ or } 1.25 \text{ db.}$$

In the case of a triangularly illuminated aperture,

$$G_n(\phi) = \frac{\sin^4(\frac{\pi}{2} \frac{L}{\lambda} \sin \phi)}{(\frac{\pi}{2} \frac{L}{\lambda} \sin \phi)^4} \approx \frac{\sin^4(\frac{\pi}{2} \frac{L}{\lambda} \phi)}{(\frac{\pi}{2} \frac{L}{\lambda} \phi)^4}$$

one obtains an improvement in signal-to-clutter of

$$I_{S/C} = \frac{8}{3} \left(\frac{76}{105}\right) = 1.93 \text{ or } 2.85 \text{ db.}$$

The higher S/C improvement for the triangular illumination is due to the fact that going from rectangular to triangular illumination broadens the antenna pattern beamwidth, which increases the original clutter level, and thereby allows more apparent "improvement." Actually, the <u>output S/C ratio</u> is the same for both patterns and is, in fact, independent of the aperture illumination, provided that the entire aperture is used. A penalty accrues to the broader beam, however, as the MCF will emphasize receiver noise more in its case.

* The approximation $\sin \phi \approx \phi$ is very good for the small values of ϕ subtended by a narrow beam antenna.

III. COMMENTS AND CONCLUSIONS

Based on the foregoing analysis, the improvement in S/C which can be achieved by processing the returns from more than one beam position is very limited, on the order of 1 or 2 db, depending on the particular antenna pattern. This sets an upper bound on the improvement in S/C for the GE monopulse technique, which effectively processes returns from two beam positions.

Let us compare the above theoretical results with the results of GE's computer simulation. GE concluded:[5]

> "...it is safe to say that at least 10-15 db of improvement in clutter rejection can be obtained, i.e., 90-97% of complex clutter pulses will not appear on the display..."

It is rather unfortunate that GE chose to describe a 90-97% reduction in false alarms ("complex clutter pulses") as a 10-15 db improvement in "clutter rejection," since the term "clutter rejection" has conventionally been used in MTI work to mean a reduction in the effective clutter level. As a matter of fact, a 90% decrease in the false alarm rate typically corresponds to only about 1/3 of a db reduction in clutter level, while a 97% decrease in the false alarm rate corresponds to only about 0.6 db reduction in clutter level (for bandwidths on the order of megacycles), as shown in Fig. 2.[6]

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[5] W. Hausz, loc. cit., p. 38.

[6] M. Skolnik, Introduction to Radar Systems, McGraw-Hill, New York, 1962, p. 31.

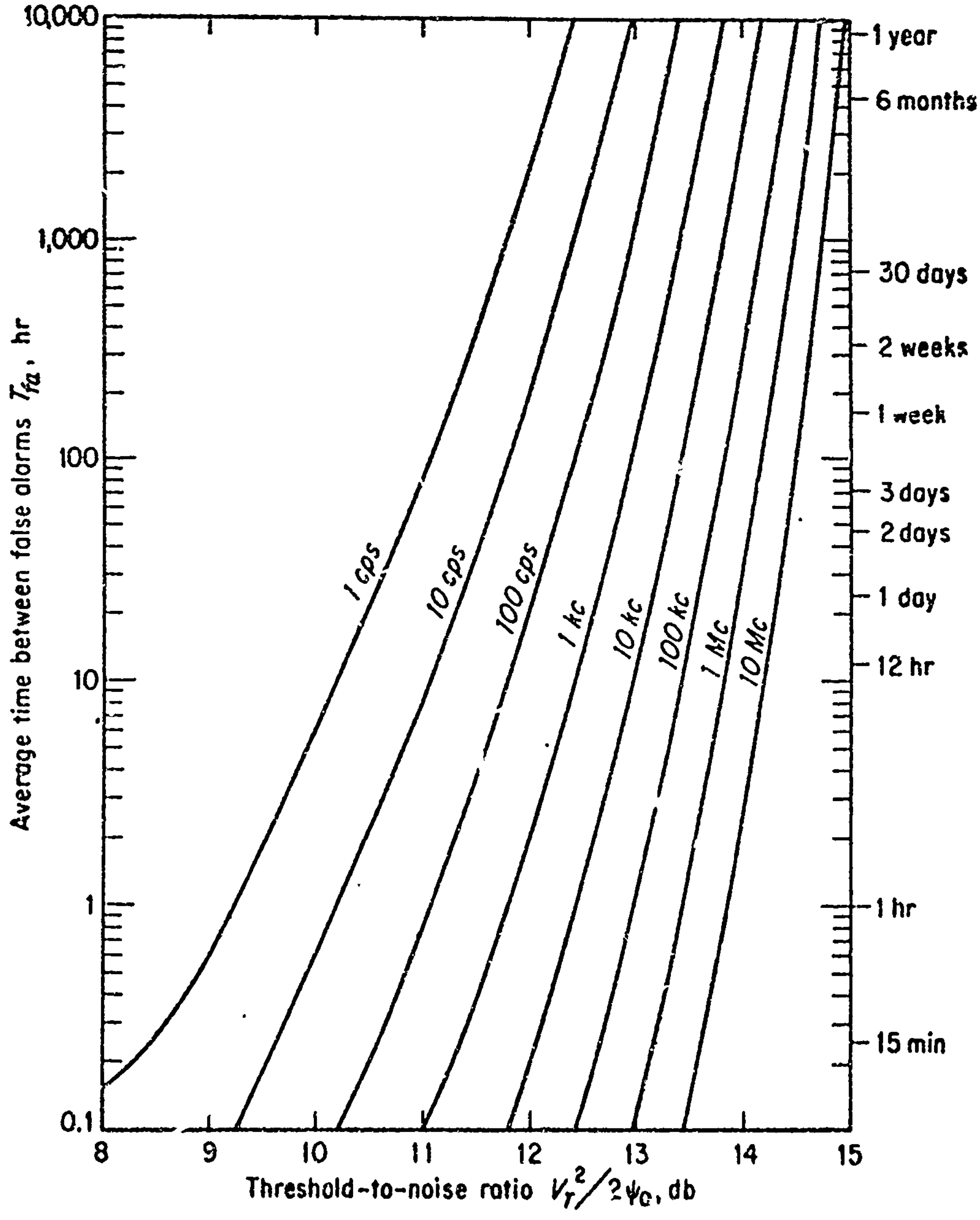

Fig. 2 Average Time Between False Alarms as a Function of the Threshold Level V_T and Receiver Bandwidth B; ψ_o is the Mean-Square Noise Voltage. (From M. Skolnik, Introduction to Radar Systems, p. 31.)

Thus, GE's technique gives not 10-15 db reduction in effective clutter level, but only about 0.3 to 0.6 db reduction. These latter values are consistent with the theoretical limit derived here of 1.25 db of clutter reduction (based on the $\sin x/x$ pattern assumed by GE).

Were the monopulse technique capable of effecting a 90-97% reduction in "discrete" clutter returns (e.g., water tanks, towers, etc.) this would be a significant accomplishment. Unfortunately, the point-like discrete clutter sources are indistinguishable from point-like targets such as aircraft, and as such will pass, unaffected, through the system.

It may be concluded, then, that only small improvements in signal-to-clutter ratio are possible by utilizing returns from more than one beam position. Based on GE's own computed results, the improvement is on the order of 0.3 db to 0.6 db. The theoretical upper-bound computed here is an improvement of 1.25 db for a $\sin x/x$ antenna pattern (assumed by GE) and 2.85 db for a $\sin^2 x/x^2$ antenna pattern. The greater apparent improvement for the $\sin^2 x/x^2$ pattern illustrates the fact that if a low sidelobe antenna design is used, with consequently broadened main beam and increased input clutter, the MCF processor allows one to obtain the same output S/C ratio as in the narrow beam case. However, a penalty accrues for the broader beam, since receiver noise will be emphasized to a greater extent by the MCF, requiring more transmitter power.

Appendix A

CORRELATION OF CLUTTER SIGNALS ON EACH HALF-ARRAY OF A PHASE-COMPARISON MONOPULSE ANTENNA

For a total array of length L, the transmit or sum beam (Σ) has a width β_Σ of

$$\beta_\Sigma = \lambda/L ,$$

where λ is RF wavelength. The azimuth extent L_c of a clutter cell at range R is

$$L_c = R\beta_\Sigma = \frac{R\lambda}{L} .$$

The clutter cell will have a random, noise-like backscatter pattern with a minimum lobe width β_c of

$$\beta_c = \lambda/L_c = \lambda \cdot \frac{L}{R\lambda} = L/R.$$

The amplitude and phase of the clutter backscatter pattern will be essentially uncorrelated for a change in angle of β_c. The distance L' subtended back at the radar array by the clutter lobe of width β_c is

$$L' = R\beta_c = R \cdot \frac{L}{R} = L .$$

Thus, two points on the array separated by a distance L receive essentially uncorrelated clutter signals. Since the phase centers of the two half-arrays of the monopulse antenna are separated by L/2, it may be concluded that the clutter signals from each half-array will exhibit appreciable, although not complete, decorrelation.

Appendix B

OPTIMUM CLUTTER REJECTION FILTER FOR A SCANNING ANTENNA

A. General

A radar discriminates in favor of point targets and against distributed clutter by using a short transmitted pulse and a narrow-beam antenna pattern.[1] An optimum clutter rejection filter for a single pulse has been derived by Urkowitz and has a transfer function which is the inverse of the spectrum of the radar pulse.[2]

The Urkowitz filter is derived from the optimum, matched filter (MF) transfer function:[3]

$$\frac{S^*(f)}{N(f)} \tag{1}$$

(where $S^*(f)$ is the conjugate of the radar pulse spectrum $S(f)$ and $N(f)$ is the noise spectrum) by observing that in the case of clutter one has

$$N(f) = |S(f)|^2. \tag{2}$$

Equation (2) follows from the fact that while individual clutter returns are of the form $S(f)$, their phases are random due to the arbitrary location of the clutter elements in range, causing the

[1] Doppler discrimination of moving point targets in clutter is not under consideration in this analysis.

[2] H. Urkowitz, loc. cit., pp. 1024-1031.

[3] G.L. Turin, "An Introduction to Matched Filters," IRE Transactions on Information Theory, Vol. IT-6, No. 3, June 1960, pp. 311-329.

clutter returns to add in mean-square fashion, or powerwise. Substituting Eq. (2) into Eq. (1) gives the optimum, "matched clutter filter" (MCF) transfer function:

$$\frac{S^*(f)}{|S(f)|^2} = \frac{S^*(f)}{S(f)S^*(f)} = \frac{1}{S(f)}, \tag{3}$$

which is seen to be the inverse of the transmitted pulse spectrum.

An analogous situation exists with respect to the angular resolution of the radar. Consider a CW (continuous wave) radar scanning its antenna in angle. The scanning antenna pattern imposes a characteristic modulation on the CW echo from every object. The received signal modulation due to antenna pattern rotation may be likened to the pulse envelope in the case of a pulsed radar. The clutter returns will add randomly, or powerwise, not only because of their random location in range, but also because of their random location in azimuth. Urkowitz's optimum clutter rejection filter approach may now be applied to filter the modulation of the CW signal and thereby optimally utilize the angular resolution capabilities of the radar to reject clutter (as contrasted to the range resolution in Urkowitz's analysis).

B. Analysis

Antenna Pattern Relationship

Given an aperture excitation function A(x) over an aperture length L, A(x) and the one-way antenna pattern (voltage), $g(\phi)$, are related by

$$g(\phi) = \int_{-L/2}^{L/2} A(x) \exp\left[j\ 2\pi \frac{x}{\lambda} \sin\phi \right] dx \tag{4}$$

$$A(x) = \frac{1}{\lambda} \int_{-\infty}^{\infty} g(\phi) \exp\left[-j\ 2\pi \frac{x}{\lambda} \sin\phi \right] d(\sin\phi) \tag{5}$$

where λ is RF wavelength and ϕ is angle off the normal to the aperture. The one-way power pattern $G(\phi)$ is

$$G(\phi) = g^2(\phi) \tag{6}$$

which is also the two-way voltage pattern. The two-way power pattern is

$$G^2(\phi) = g^4(\phi) \tag{7}$$

<u>Input Clutter Power</u>

Let there be ρ clutter elements per radian (average), each producing a received power of σ (average) based on unity antenna gain. The total (average) input clutter power $\left(P_c\right)_i$ received on an antenna with two-way power pattern $G^2(\phi)$ is

$$\left(P_c\right)_i = \int_{-\infty}^{\infty} G^2(\phi)(\sigma\rho d\phi) = \sigma\rho \int_{-\infty}^{\infty} G^2(\phi) d\phi, \tag{8}$$

whether the antenna is stationary or rotating.*

<u>Input Clutter Power Spectrum</u>

Let the input signal received by the rotating antenna from a single clutter element returning <u>unit</u> power (at unity antenna gain) be denoted s(t). Assuming a one ohm antenna load,

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*Because the antenna patterns of interest here are narrow beams, the limits of integration may be taken as $-\infty$ to $+\infty$, rather than $-\pi/2$ to $+\pi/2$, without significant error.

the received signal is the two-way voltage pattern or one-way power pattern:

$$s(t) = g^2\{\phi(t)\} = G\{\phi(t)\} \tag{9}$$

where

$$\phi = \omega_r t \tag{10}$$

and ω_r is the angular rotation rate of the antenna (radians/sec).

Let S(f) be the Fourier, or voltage, spectrum of s(t). Because the random phase of the clutter elements causes them to add on a mean-square basis, the input power spectrum of the clutter, $\left[p_c(f)\right]_i$, is obtained directly as

$$\left[p_c(f)\right]_i = K_c|S(f)|^2, \tag{11}$$

where K_c is a proportionality constant. K_c is evaluated as follows. Total input clutter power $\left(P_c\right)_i$ is obtained by integrating Eq. (11) over all frequencies:

$$\left(P_c\right)_i = K_c \int_{-\infty}^{\infty} |S(f)|^2 df. \tag{12}$$

The integral in Eq. (12) is the energy of the input signal waveform. This is equal to the integral over time of the squared input signal, or

$$\int_{-\infty}^{\infty} |S(f)|^2 df = \int_{-\infty}^{\infty} s^2(t)\,dt = \int_{-\infty}^{\infty} G^2\{\phi(t)\}dt = \frac{1}{\omega_r}\int_{-\infty}^{\infty} G^2(\phi)\,d\phi, \tag{13}$$

using Eqs. (9) and (10). Substituting Eqs. (8) and (13) into (12) gives

$$K_c = \frac{\left(P_c\right)_i}{\int_{-\infty}^{\infty} |S(f)|^2 df} = \frac{\sigma\rho \int_{-\infty}^{\infty} G^2(\phi)\, d\phi}{\frac{1}{\omega_r} \int_{-\infty}^{\infty} G^2(\phi)\, d\phi} = \omega_r \sigma\rho \qquad (14)$$

Substituting the above value of K_c in Eq. (11) gives the input clutter power spectrum

$$\left[p_c(f)\right]_i = \omega_r \sigma\rho |S(f)|^2 \qquad (15)$$

Output Clutter Power Spectrum

For the matched clutter filter, $1/S(f)$, of Eq. (3) and the input clutter power spectrum of Eq. (15), the spectrum $\left[p_c(f)\right]_o$ of the clutter out of the MCF is

$$\left[p_c(f)\right]_o = \frac{p_c(f)_i}{|S(f)|^2} = \frac{\omega_r \rho\sigma |S(f)|^2}{|S(f)|^2} = \omega_r \sigma\rho = K_c \qquad (16)$$

The output clutter spectrum is seen to be flat.

Output Clutter Bandwidth

Clutter has the same bandwidth as the received signal waveform $s_i(t)$ of Eq. (9):

$$s(t) = g^2\{\phi(t)\}. \qquad (17)$$

Consider first the signal

$$s'(t) = g\{\phi(t)\} \qquad (18)$$

and its spectrum $S'(f)$:

$$S'(f) = \int_{-\infty}^{\infty} s'(t) \exp\left[-j\, 2\pi\, ft\right] dt = \int_{-\infty}^{\infty} g\{\phi(t)\} \exp\left[-j\, 2\pi\, ft\right] dt$$

$$= \frac{1}{\omega_r} \int_{-\infty}^{\infty} g(\phi) \exp\left[-j\, 2\pi \frac{f}{\omega_r} \phi\right] d\phi \qquad (19)$$

using the substitution $\phi = \omega_r t$. Comparing Eqs. (19) and (5), the two expressions are seen to be identical in form except for the difference between $\sin\phi$ and ϕ. For the narrow beam antennas under consideration here, the approximation

$$\sin\phi \approx \phi \qquad (20)$$

is valid, allowing one to make the statement that f and ω_r in Eq. (19) are analogous to x and λ, respectively, in Eq. (5). It follows that the bandwidth B' of s(t) has the same ratio to ω_r as the aperture length L has to λ, or:

$$\frac{B'}{\omega_r} = \frac{L}{\lambda} \qquad (21)$$

Solving for B' gives

$$B' = (L/\lambda)\omega_r \qquad (22)$$

as the bandwidth of s'(t).

We are actually interested in the bandwidth B of s(t), the square of s'(t). Since squaring the received signal will double its bandwidth, one obtains

$$B = 2B' = 2(L/\lambda)\omega_r \qquad (23)$$

Output Clutter Power

The output power $\left(P_c\right)_o$ is simply the constant spectral density of Eq. (16) multiplied by the bandwidth of Eq. (23):

$$\left(P_c\right)_o = K_c B = 2\sigma\rho(L/\lambda)\omega_r^2 \tag{24}$$

Peak Output Signal

Assume the received power from a point target is σ_t (at unity antenna gain). The spectrum of the target return is

$$\sqrt{\sigma_t}\, S(f) \tag{25}$$

(where $\sqrt{\sigma_t}$ is the voltage developed across the assumed one ohm load). The effect of putting the spectrum of Eq. (25) through the matched clutter filter $1/S(f)$ of Eq. (3) is to produce a flat output spectrum of amplitude

$$\sqrt{\sigma_t}\, S(f) \cdot \frac{1}{S(f)} = \sqrt{\sigma_t} \tag{26}$$

over the bandwidth B.

The target output voltage $\left[s_t(t)\right]_o$ is the Fourier transform of the spectrum of Eq. (26), or

$$\left[s_t(t)\right]_o = \int_{-B/2}^{B/2} \sqrt{\sigma_t}\, \exp\left[j\, 2\pi\, ft\right] df = \sqrt{\sigma_t}\, B\left(\frac{\sin \pi Bt}{\pi Bt}\right) \tag{27}$$

having a peak voltage of $\sqrt{\sigma_t}\, B$ and a peak power, $\left(P_s\right)_o$, of

$$\left(P_s\right)_o = \sigma_t B^2 \tag{28}$$

Since target and clutter spectra are both limited to bandwidth B, the matched clutter filter response should not extend outside the bandwidth B, in order to keep receiver noise of thermal origin to a minimum.

Output Signal-to-Clutter Ratio

The ratio of peak signal (target) output power to output clutter power, from Eqs. (28) and (24) is

$$\frac{(P_s)_o}{(P_c)_o} = \frac{\sigma_t B^2}{2\sigma\rho(L/\lambda)\omega_r^2} \tag{29}$$

Substituting the value of B from Eq. (23) into (29) gives the S/C ratio out of the matched clutter filters:

$$\frac{(P_s)_o}{(P_c)_o} = 2\frac{\sigma_t}{\sigma\rho}(L/\lambda). \tag{30}$$

Note that $(P_s)_o / (P_c)_o$ is independent of the antenna aperture excitation provided that it extends over the full aperture L.

Signal-to-Clutter Ratio with No Filtering

With no filtering, the input clutter power is given by Eq. (8), while the input signal power $(P_s)_i$ is

$$(P_s)_i = \sigma_t G^2(0) \tag{31}$$

when the target is at the maximum of the antenna pattern. Taking the ratio of Eq. (31) to (8) gives

$$\frac{(P_s)_i}{(P_c)_i} = \frac{\sigma_t G^2(0)}{\sigma\rho \int_{-\infty}^{\infty} G^2(\phi)\,d\phi}$$

$$= \frac{\sigma_t G^2(0)}{\sigma\rho G^2(0) \int_{-\infty}^{\infty} G_n^2(\phi)\,d\phi} = \frac{\sigma_t}{\sigma\rho}\,\frac{1}{\int_{-\infty}^{\infty} G_n^2(\phi)\,d\phi} \tag{32}$$

where $G_n(\phi)$ is the normalized one-way power pattern of the antenna.

Signal-to-Clutter Improvement

The signal-to-clutter improvement $I_{S/C}$ due to matched clutter filtering is the ratio of Eq. (30) to (32):

$$I_{S/C} = \frac{(P_s)_o / (P_c)_o}{(P_s)_i / (P_c)_i} = 2(L/\lambda) \int_{-\infty}^{\infty} G_n^2(\phi)\, d\phi. \qquad (33)$$

For a uniformly illuminated antenna aperture of length L, the normalized one-way power pattern is

$$G_n(\phi) = \frac{\sin^2(\pi \frac{L}{\lambda} \sin \phi)}{(\pi \frac{L}{\lambda} \sin \phi)^2} \approx \frac{\sin^2(\pi \frac{L}{\lambda} \phi)}{(\pi \frac{L}{\lambda} \phi)^2} \qquad (34)$$

giving an S/C improvement of

$$I_{S/C} = 2(\frac{L}{\lambda}) \int_{-\infty}^{\infty} \frac{\sin^4(\pi \frac{L}{\lambda} \phi)}{(\pi \frac{L}{\lambda} \phi)^4} d\phi = \frac{4}{3} \text{ or } 1.25 \text{ db} \qquad (35)$$

For a triangularly illuminated antenna aperture of length L, the normalized one-way power pattern is

$$G_n(\phi) = \frac{\sin^4(\frac{\pi}{2} \frac{L}{\lambda} \sin \phi)}{(\frac{\pi}{2} \frac{L}{\lambda} \sin \phi)^4} \approx \frac{\sin^4(\frac{\pi}{2} \frac{L}{\lambda} \phi)}{(\frac{\pi}{2} \frac{L}{\lambda} \phi)^4}, \qquad (36)$$

giving a S/C improvement of

$$I_{S/C} = 2(\frac{L}{\lambda}) \int_{-\infty}^{\infty} \frac{\sin^8(\frac{\pi}{2} \frac{L}{\lambda} \phi)}{(\frac{\pi}{2} \frac{L}{\lambda} \phi)^8} = \frac{8}{3}(\frac{76}{105}) = 1.93 \text{ or } 2.85 \text{ db}. \qquad (37)$$